

Astrophysical Naturalness

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ABSTRACT

I suggest that stars introduce mass and density scales that lead to ‘naturalness’ in the Universe. Namely, two ratios of order unity. (1) The combination of the stellar mass scale, $M_*(c, \hbar, G, m_p, m_e, e, \dots)$, with the Planck mass, M_{Pl} , and the Chandrasekhar mass leads to a ratio of order unity that reads $N_{\text{Pl}*} \equiv M_{\text{Pl}}/(M_* m_p^2)^{1/3} \simeq 0.15 - 3$, where m_p is the proton mass. (2) The ratio of the density scale, $\rho_{\text{D}*}(c, \hbar, G, m_p, m_e, e, \dots) \equiv (G \tau_{\text{nuc}*}^2)^{-1}$, introduced by the nuclear life time of stars, $\tau_{\text{nuc}*}$, to the density of the dark energy, ρ_Λ , is $N_{\Lambda*} = \rho_\Lambda/\rho_{\text{D}*} \approx 10^{-7} - 10^5$. Although the range is large, it is critically much smaller than the 123 orders of magnitude usually referred to when ρ_Λ is compared to the Planck density. In the pure fundamental particles domain there is no naturalness; either naturalness does not exist or there is a need for a new physics or new particles. The ‘Astrophysical Naturalness’ offers a third possibility: stars introduce the combinations of, or relations among, known fundamental quantities that lead to naturalness.

1. Introduction

The naturalness topic is nicely summarized by Natalie Wolchover in an article from May 2013 in Quanta Magazine¹. I here discuss two points as listed in the talk “Where are we heading?” given by Nathan Seiberg in 2013:² (1) “Why doesn’t dimensional analysis work? All dimensionless numbers should be of order one”; (2) “The cosmological constant is quartically divergent - it is fine tuned to 120 decimal points.”

My answer to the first point is that in astrophysics dimensional analysis does work when stars are considered as fundamental entities. This answers the second question as well. If the

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¹<https://www.quantamagazine.org/20130524-is-nature-unnatural/>

²<http://physicsforme.com/2013/07/24/nathan-seiberg-where-are-we-heading/>

nuclear lifetime of stars is taken to be a dynamical time of the Universe, then naturalness emerges from the observed cosmological constant. No fine tuning is required.

Many relations among microscopic quantities and their relations with macroscopic quantities are discussed by Carr & Rees (1979) who try to explain these relations, or else they refer to the anthropic principle to account for some of the relations. A more recent study was conducted by Burrows & Ostriker (2014). Here I do not repeat the explanations in those two papers. I simply take stars to provide the relations among the many physical constants and particles properties, and show that naturalness emerges from the relations introduced by stars. One might refer to relations introduced by stars as coincidental (e.g., Carr & Rees 1979), but in the present essay I prefer to refer to these relations as naturalness. My goal is to suggest a third option to treat naturalness, as explained in the last section.

This essay does not discover anything new, but rather suggests to include stars as ‘fundamental entities’ when considering naturalness in our Universe. As naturalness was discussed in talks and popular articles, I use them as references. I also limit the discussion to two commonly discussed quantities in relation to naturalness, the Planck mass and the cosmological constant (dark energy). Many other relations and coincidences can be found in the papers by Carr & Rees (1979) and Burrows & Ostriker (2014). I will not touch the question of multiverse which is often connected to the values of fundamental quantities, e.g., Weinberg (2005) and Livio & Rees (2005).

2. The Chandrasekhar mass

The Planck mass that starts the discussion on naturalness is defined as

$$M_{\text{Pl}} = \left(\frac{\hbar c}{G} \right)^{1/2} = 2.177 \times 10^{-5} \text{ g}. \quad (1)$$

It is many orders of magnitude above the mass of the Higgs boson and all other fundamental particles. If one constrains himself to the particle world, no naturalness exists (e.g. Dine 2015). Let us add stars.

We start with the Chandrasekhar mass limit M_{Ch} . This is the maximum mass where a degenerate electron gas can support a body against gravity. The electrons are relativistic at this mass limit, and the expression reads

$$M_{\text{Ch}} = K_1 \left(\frac{Z}{A} \right)^2 \left(\frac{M_{\text{Pl}}}{m_p} \right)^2 M_{\text{Pl}} = K_1 \left(\frac{Z}{A} \right)^2 M_{\text{BCh}}, \quad (2)$$

where $m_p = 1.673 \times 10^{-24} \text{ g}$ is the proton mass, and Z and A are the atomic number and atomic mass number, respectively, of the element(s) composing the white dwarf (the ratio

Z/A is the mean number of electrons per nucleon in the white dwarf). The constant $K_1 \simeq 3.1$ is composed of pure numbers (no physical constants), and $K_1(Z/A)^2 \simeq 0.8$ for white dwarfs in nature where $Z = 0.5A$. The last equality defines what I term the bare Chandrasekhar mass

$$M_{\text{BCh}} \equiv \left(\frac{M_{\text{Pl}}}{m_p} \right)^2 M_{\text{Pl}} = \alpha_G^{-1} M_{\text{Pl}} = 1.85 M_{\odot}, \quad (3)$$

where $\alpha_G = Gm_p^2/\hbar c = 5.9 \times 10^{-39}$ is the gravitational fine structure constant, which is also used to express M_{BCh} , e.g., Carr & Rees (1979).

3. Naturalness with stars

The mass of stars is determined by the requirement that hydrogen burns to helium. From below it is limited by brown dwarfs, where the star cannot compress and heat enough to ignite hydrogen. The minimum mass for a star is $M_* > 0.08 M_{\odot}$. The maximum stellar mass of hundreds solar masses is not well determined, but radiation pressure limits the upper mass (e.g., Carr & Rees 1979; Burrows & Ostriker 2014). Interestingly, the Chandrasekhar mass sits more or less in the center of the stellar mass range in logarithmic scale (e.g., Carr & Rees 1979)

$$N_{\text{M}*} \equiv \frac{M_{\text{BCh}}}{M_*} = \left(\frac{M_{\text{Pl}}}{m_p} \right)^2 \frac{M_{\text{Pl}}}{M_*} \simeq 0.01 - 20. \quad (4)$$

In the logarithmic scale the range of this ratio is approximately -2 to 1.4 , much-much smaller than the 17 orders of magnitude difference between the mass of the Higgs boson and the Planck mass. Moreover, if the ratio is with the Planck mass rather than M_{BCh} , then the ratio is closer to unity, as it reads

$$N_{\text{Pl}*} \equiv \frac{M_{\text{Pl}}}{(M_* m_p^2)^{1/3}} \simeq 0.15 - 3. \quad (5)$$

It is important to emphasize that the mass of stars is determined by the requirement that hydrogen experience thermonuclear burning to helium. The Chandrasekhar mass is determined from the pressure that a degenerate electrons gas can hold against gravity. Nothing demands them to be equal. But they are. Namely, the ratio of the Chandrasekhar mass, that is composed of the Planck and the proton masses, to stellar mass is or order unity. *This is naturalness.*

Of course, the properties of stars are determined by the properties of the four fundamental forces, as all of them are involved in the nuclear burning and stellar structure, and the properties of the particles involved. The question is what combination of the fundamental

constants of the forces and of the particles’ properties gives two quantities whose ratio is ≈ 1 ? The answer here is that stars form this combination as

$$M_* = M_*(c, \hbar, G, m_p, m_e, e, \text{Forces of nature}, \dots), \quad (6)$$

hence give us the naturalness in the Universe. This is expressed in equations (4) and (5).

In other words, much as the proton ‘forms’ a combination from the properties of the quarks and the electric and color forces to give a mass, the proton mass m_p , so do stars. But stars build a much more complicated combination, and with many more, of the fundamental constants and forces, and the output of this relation is not quantized, but it is rather a continuous function.

I note that Carr & Rees (1979) try to show that $N_{\text{Pl}*} \sim 1$ is expected. However, they had to use numbers from more complicated calculations than just order of magnitude estimates. They specifically use the nuclear burning temperature of hydrogen, T_H , and take a factor of $q \sim 10^{-2}$ in the expression $kT_H = qm_e c^2$. Burrows & Ostriker (2014) consider the ratio between the maximum stellar mass and the Chandrasekhar mass, and take the extra (external) factor that comes from observations and detailed calculations to be the ideal gas pressure to total pressure ratio β . In setting the lower stellar mass limit Burrows & Ostriker (2014) take another extra parameter to get the burning temperature of hydrogen. The parameter is the ratio of the Gamow energy to $k_B T/3$, which they set equal to 5. That is, it is not trivial to express stellar properties from fundamental particles and physical constants. My approach is different. I avoid these extra parameters. I take stars to simply provide the relations among the different quantities.

There is also the demand that the baryonic density in the Universe be high enough for stars to form in the first place (e.g., Carr & Rees 1979; Livio & Rees 2005). A related natural ratio is discussed in section 5

4. Stellar explosion energy

The naturalness has several implications. One of them is that regular stars can lead to white dwarfs with a mass close to and above the Chandrasekhar mass. White dwarfs with that mass or above, and iron cores of massive stars with that mass, explode eventually as a supernova. White dwarfs explode as thermonuclear supernovae where carbon and oxygen burn to nickel; cores of massive stars explode as core-collapse supernovae where a neutron star is formed. The typical kinetic energy of the ejected gas in supernovae, $\approx 10^{51}$ erg, can be derived from fundamental quantities.

The radius of an idealized white dwarf supported by a degenerate non-relativistic electrons gas is given by

$$R_{\text{WD}} = K_2 \frac{\hbar^2}{G m_e m_p^{5/3} M_{\text{WD}}^{1/3}} \left(\frac{Z}{A} \right)^{5/3}, \quad (7)$$

where M_{WD} is the white dwarf mass and the constant $K_2 \approx 1$ is composed of pure numbers. For other forms of this expression for the white dwarf radius see Burrows & Ostriker (2014). Although for a white dwarf at the Chandrasekhar mass the electrons gas is degenerate, I nonetheless substitute the bare Chandrasekhar mass M_{BCh} in equation (7) to obtain an estimate for the bare white dwarf radius

$$R_{\text{BWD}} \equiv \frac{\hbar^2}{G m_e m_p^{5/3} M_{\text{BCh}}^{1/3}} = \frac{G}{m_e c^2} \frac{M_{\text{Pl}}^3}{m_p} = 5000 \text{ km}. \quad (8)$$

Due to the factor $(Z/A)^{5/3}$ the real radius is smaller by a factor of ≈ 3 . We can define the bare gravitational-energy of the bare white dwarf as

$$E_{\text{BCh}} \equiv \frac{G M_{\text{BCh}}^2}{R_{\text{BWD}}} = \left(\frac{M_{\text{Pl}}}{m_p} \right)^3 m_e c^2 = 1.8 \times 10^{51} \text{ erg}. \quad (9)$$

This is the typical kinetic energy of the mass ejected in supernova explosions, either massive stars (core collapse supernovae) or white dwarfs (Type Ia supernovae). Simply the explosion energy is of the order of the binding energy.

Accurate calculations give lower binding energy values to exploding white dwarfs and collapsing cores by a factor of several. This is because the internal energy has a positive value. The explosion kinetic energy is then several times the binding energy. But this does not change the argument.

The factor $(M_{\text{Pl}}/m_p)^3$ is the number of nucleons in the white dwarf, so that the binding energy per nucleon is $\approx m_e c^2$. This is also the order of magnitude of the nuclear energy released per nucleon when carbon and oxygen burn to nickel. This nuclear energy is the energy source of type Ia supernovae. Indeed, about 20 – 60% of the white dwarf burns to nickel during a type Ia supernova.

When a core of a massive star collapses to a neutron star it releases a total energy of $\approx \text{few} \times 10^{53}$ erg. This energy comes from the final radius of the neutron star which is determined from nuclear repulsive forces acting against gravity. Most of this energy is carried out by neutrinos (and anti-neutrinos) of the three kinds.

5. The naturalness of dark energy

The usual approach to search for naturalness is to compare the observed density of the dark energy $\rho_\Lambda = 7 \times 10^{-30} \text{ g cm}^{-3}$ with the Planck density $\rho_{\text{Pl}}(c, \hbar, G) = M_{\text{Pl}} l_{\text{Pl}}^{-3} = 5.155 \times 10^{93} \text{ g cm}^{-3}$, where $l_{\text{Pl}} = (\hbar G/c^3)^{1/2}$ is the Planck length. This gives an ‘unnatural’ ratio of $U = \rho_\Lambda/\rho_{\text{Pl}}(c, \hbar, G) = 10^{123}$. We in astrophysics are not accustomed to such astronomical numbers. This ‘unnatural’ ratio is referred to as the cosmological constant problem (e.g., Carroll 2002). A density function to replace $M_{\text{Pl}} l_{\text{Pl}}^{-3}$ is required.

As we saw in previous sections, stars introduce a (complicated) combination of the fundamental quantities to give a mass ratio of order unity, that is, an astrophysical mass naturalness (equation 4). Stars also introduce some typical time scales, like their dynamical time scale, thermal time scale, and nuclear life time. I take here the nuclear time scale which is the life time over which a star evolves, as I compare the quantity with the dark energy that is related to the evolution of the Universe.

Stars spend most of their nuclear lives burning hydrogen to helium. The nuclear life time of stars depends mainly on the initial mass of the star, with $\tau_{\text{nuc}*}(0.1M_\odot) \approx 10^{13} \text{ yr}$, $\tau_{\text{nuc}*}(M_{\text{BCh}}) \approx 10^9 \text{ yr}$, and $\tau_{\text{nuc}*}(M \gtrsim 10M_\odot) \approx 10^7 \text{ yr}$. I now equate this nuclear time scale to a dynamical time scale $t_{D*} \equiv \tau_{\text{nuc}*} = (G\rho_{D*})^{-1/2}$ associated with a density

$$\rho_{D*} \equiv (G \tau_{\text{nuc}*}^2)^{-1} \approx 10^{-34} - 10^{-22} \text{ g cm}^{-3}. \quad (10)$$

This density comes for the nuclear life time of stars that depends on many fundamental parameters. Namely,

$$\rho_{D*} = \rho_{D*}(c, \hbar, G, m_p, m_e, e, \text{Forces of nature}, \dots) \quad (11)$$

The point here is that stars introduce the basic relation among these fundamental quantities.

The second natural number defined in this essay is therefore

$$N_{\lambda*} \equiv \frac{\rho_\Lambda}{\rho_{D*}} \approx 10^{-7} - 10^5. \quad (12)$$

Although the range is large, it is critically much smaller than the 123 orders of magnitude usually referred to when ρ_Λ is compered to ‘natural density’. Moreover, a ratio of unity sits just near the center of this range.

I conclude that stars introduce a nuclear time scale, whose associated dynamical time scale leads to a density about equal to the dark energy density. Again, the nuclear time scale of stars is determined by a complicated relation of fundamental quantities, constants and particle properties. Stars combine the fundamental quantities to lead to a naturalness.

It is important to emphasize that the approach here is different than the question “Why does the cosmological constant (dark energy) became significant only recently? (e.g., Livio & Rees 2005). Namely, why the age of the universe is about equal to the dynamical time associated with the density of the cosmological density? The approach here also differs from coincidental identities that are related to the present age or size of the Universe (e.g., Carr & Rees 1979).

In the present approach the age of the universe has no importance at all. The same argument presented here holds as soon as hydrogen becomes the main element in the universe; the first minute of the universe, at an age of 10^{-16} times the present Universe age. The same argument will be true when the universe be 10^{16} times its present age (as long as the dark energy density stays constant).

It is true that if the cosmological constant (dark energy) had been much larger, stars would not have formed (see, e.g., Garriga et al. 2000, and also for other time scales involving the cosmological constant). The value of primordial density fluctuations is also related to the question of star formation (Livio & Rees 2005). But here I don’t examine these questions; I look for ratios of order unity, i.e., naturalness.

6. Summary

The naturalness question I studied here can be posed as follows: “What is the combination of the fundamental constants and particle properties that leads to a ratio of two values that is of order unity?” In the present essay I showed that stars introduce these combinations that give what might be termed “Astrophysical Naturalness”.

Stars introduce the stellar mass given in equation (6) that leads to the natural relation (4), or (5). Stars also introduce a nuclear timescale. If this time scale is associated with a dynamical time scale, then a density ρ_{D*} given by a very complicated relation (eq. 11) is defined. The density leads to the natural relation (12).

Nathan Seiberg summarizes his talk by a diagram that leaves two basic options, (i) abandon naturalness, or (ii) go beyond known physics/particles to find naturalness. Here I take a third option which is basically to add stars as a basic entity in our Universe, much as the proton is a composite particle. This brings out naturalness in a beautiful way, at least in the eyes of an astrophysicist.

The arguments presented here are not the anthropic principle, e.g., as presented by Livio & Rees (2005). Livio & Rees (2005) list the necessity of stars to form in order to have

life. I differ here in two respects. (1) I treat stars on the same level as I treat baryons. I do not require that the properties of protons allow stars as Livio & Rees (2005) do. I simply treat stars as I treat baryons (although definitely stars are more complicated and composed of baryons). Both baryons and stars are composite entities that exist in the Universe. They appear on the same level in equations (4) and (5). (2) The arguments presented here do not require the presence of carbon in the universe. All arguments here apply if nuclear reactions would have ended with helium. As well, life requires some chemical properties. The arguments presented here don't involve chemistry at all. For example, even if all stars were much hotter and the strong UV radiation would prevent life, the arguments presented here still hold.

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